Linear estimation of 4-D illumination light field from diffuse reflections

Takahito Aoto*, Tomokazu Sato*, Yasuhiro Mukaigawa†and Naokazu Yokoya*

* Nara Institute of Science and Technology, Japan

{takahito-a, tomoka-s, yokoya}@is.naist.jp

† ISIR, Osaka University, Japan
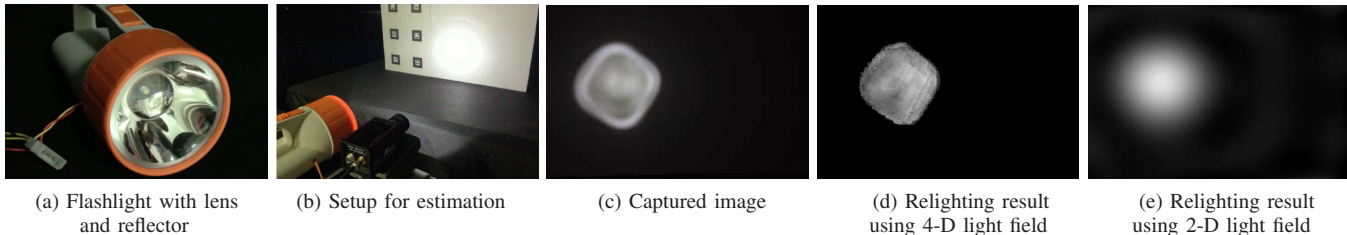
mukaigaw@am.sanken.osaka-u.ac.jp

(a) Flashlight with lens and reflector   (b) Setup for estimation   (c) Captured image   (d) Relighting result using 4-D light field   (e) Relighting result using 2-D light field

Fig. 1.   Experimental setup and comparison of experimental results.

Abstract—This paper proposes a linear estimation method for 4-D illumination light field using an inverse lighting framework. Although lighting environments have conventionally been modeled by the combination of point light sources or a spherical light field map, complex light sources such as an ordinary flashlight with lenses and reflectors, can not be correctly modeled by these approaches. In order to correctly model these complex light sources, we employ 4-D illumination light field. Unlike conventional work, we decompose observed intensities on a diffuse-reflection board into intensities of 4-D light rays by solving a linear system. We validate the effectiveness of the proposed method through the experiments on real and virtual environments.

I. INTRODUCTION

Estimation of illumination is useful for many applications, e.g. photorealistic image synthesis, photometric stereo and BRDF estimation. For this purpose, conventional methods often employ the point light source model (Fig. 2 (a)) for its simplicity [1], [2]. However, as shown in Fig. 1, the methods using these simple illumination models do not work for complex light sources including an ordinary flashlight with lenses and reflectors, due to its simplicity.

One simple extension is to allow different radiance for a different direction from a single point light source or for a single observation point (see Fig. 2 (b)). In this case, radiance in different direction is represented as a 2-D map on a sphere (here, we call this model as a 2-D light field) [3], [4]. By using the 2-D light field, we can accurately model a light source with strong directivity and a certain volume that cannot be ignored e.g. LED. This model is also useful for modeling complex lighting environments for a certain 3-D position.

However, in order to completely model the lighting environments, the 4-D light field (see Fig. 2 (c)) is necessary. Conventional approaches using the 4-D light field model suffer from the cost for measuring all rays. One straightforward
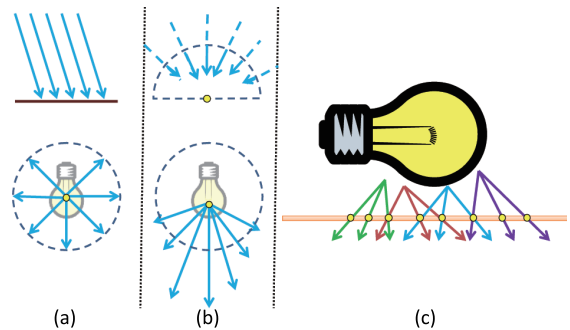

Fig. 2.   Illumination models. a) parallel light source and isotropic point light source, b) 2-D light field, c) 4-D light field.

method for measuring the 4-D light field is to capture images for all the directions from all 3-D positions. Actually, there exist many researches that directly capture a huge number of images using a camera mounted on a robot arm [5]–[8]. In order to reduce the cost for measurement, the mirrors [9], lens array [10], and filter [11] are used in the literatures. One of the problems in these direct methods is the difficulty in estimation of missing rays which have not captured in the images. On the other hand, inverse lighting methods estimate the light field by analyzing captured images [2], [12]. In these methods, direct observation of a reference object is unnecessary; instead, they back-trace the rays from the camera to the light source using the known shape of the reference object and its reflection parameters. Although many methods for inverse lighting have been proposed in recent years, simple illumination models are employed in most of these works [2]. To the best of our knowledge, there exists no work that estimate the 4-D light field using an inverse lighting framework. In this paper, we show that radiance parameters of the 4-D light field emitted from one complex light source can be estimated using an inverse lighting framework by solving

a linear system. However, the naive approach has a drawback in computational cost and stability. We propose a solution for these problems using the real spherical harmonics function and visibility masks.

II. BASIC IDEA

In this paper, we estimate a 4-D light field for one complex light source using multiple images in which an illuminated diffuse reflection board is captured from various positions, while camera position and light position are fixed. Figure 3 illustrates a scene where light rays pass through virtual plane $\mathcal{L}$ and hit diffuse-reflection board $\mathcal{M}$. Here, 4-D light field is modeled as the intensities of rays defined by the parameters of position (u, v) and direction (θ, ϕ). In this representation, irradiance $i(\boldsymbol{X})$ in the local region $\boldsymbol{X}$ on $\mathcal{M}$ is proportional to the integral of all intensities of rays that hit $\boldsymbol{X}$ as follows:

$$
\begin{aligned}
i(\boldsymbol{X}) &= \int a_{\boldsymbol{X}}(\boldsymbol{j})s(\boldsymbol{j})d\boldsymbol{j}, \\
\boldsymbol{j} &= (u, v, \theta, \phi)^{\mathrm{T}},
\end{aligned}
\tag{1}
$$

where

$$
a_{\boldsymbol{X}}(\boldsymbol{j}) = \begin{cases} \rho & ; \text{if ray } \boldsymbol{j} \text{ hits } \boldsymbol{X} \\ 0 & ; \text{otherwise} \end{cases}.
\tag{2}
$$

$s(\boldsymbol{j})$ is intensity of ray $\boldsymbol{j}$ and ρ is reflectance of $\mathcal{M}$. It should be noted that actual irradiance $i(\boldsymbol{X})$ is represented as a weighted sum of intensities of rays, since 4-D light field is discretized with parameters (u, v, θ, ϕ). The relationship between observations and 4-D light field is given. By concatenating n observations of $i(\boldsymbol{X}_n)$, the following equation is derived from Eqs. (1) and (2) as a linear system,

$$
\boldsymbol{i} = \boldsymbol{A}\boldsymbol{s},
\tag{3}
$$

where,

$$
\begin{aligned}
\boldsymbol{i} &= (i(\boldsymbol{X}_1), i(\boldsymbol{X}_2), \ldots, i(\boldsymbol{X}_n))^{\mathrm{T}} && \in \mathbb{R}^n, \\
\boldsymbol{s} &= (s(\boldsymbol{j}_1), s(\boldsymbol{j}_2), \ldots, s(\boldsymbol{j}_m))^{\mathrm{T}} && \in \mathbb{R}^m, \\
\boldsymbol{A} &= \begin{pmatrix} a_{\boldsymbol{X}_1}(\boldsymbol{j}_1), & \cdots, & a_{\boldsymbol{X}_1}(\boldsymbol{j}_m) \\ a_{\boldsymbol{X}_2}(\boldsymbol{j}_1), & \cdots, & a_{\boldsymbol{X}_2}(\boldsymbol{j}_m) \\ \vdots & \ddots & \vdots \\ a_{\boldsymbol{X}_n}(\boldsymbol{j}_1), & \cdots, & a_{\boldsymbol{X}_n}(\boldsymbol{j}_m) \end{pmatrix}.
\end{aligned}
$$

Because all parameters for $\boldsymbol{A}$ and $\boldsymbol{i}$ can be determined from the observations, theoretically, we can linearly solve this system by the least squares method if the number of n observed intensities is larger than that of m unknown parameters.

III. REDUCTION OF SYSTEM

In practice, the size of $\boldsymbol{A}$ is too large to be solved due to the large number of unknown parameters. In order to reduce the size of the linear system in Eq. (3), in this paper, we employ the real spherical harmonics function and the visibility constraint. First, in order to reduce the number of unknown parameters, radiance intensities emitted from a position (u, v) on virtual plane $\mathcal{L}$ are represented using the real spherical harmonics function. More specifically, s for the ray from local region $\boldsymbol{X}$
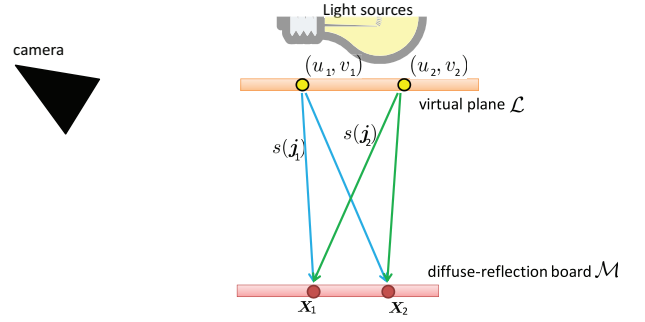
Radiance $i(\boldsymbol{X}_n)$ is observed at the points of $\boldsymbol{X}_n$ as a sum of $s(\boldsymbol{j})$

Fig. 3. Light rays that hit a diffuse-reflection board.

is represented by a weighted sum of the bases of real spherical harmonics function, which is represented by

$$
s(\theta, \phi) = \sum_{l=0}^{L} \sum_{m=-l}^{+l} c_{l,m} y_{l,m}(\theta, \phi).
\tag{4}
$$

Here, there exist $k = (L+1)^2$ different weight parameters $c_{l,m}$ and they are unknown parameters. Representable directivity of rays depends on the number of k parameters. It should be noted that although more complex light field can be reconstructed as k increases, the size of the system also becomes larger.

In order to achieve efficient estimation using a limited number of unknown parameters, we introduce visibility mask for rays. In this system, a large number of rays actually do not affect observed intensities on the camera due to two reasons: (1) ray does not hit the board, (2) ray hits the board at outside of the view. These rays can be easily determined and masked in our system by detecting dark regions on the captured images.

By introducing the real spherical harmonics function and visibility constraint, Eq. (4) can be transformed as follows:

$$
\boldsymbol{i} = \boldsymbol{Y}\boldsymbol{c},
\tag{5}
$$

$$
\boldsymbol{c} = (\boldsymbol{c}_1^{\mathrm{T}}, \boldsymbol{c}_2^{\mathrm{T}}, \ldots, \boldsymbol{c}_t^{\mathrm{T}})^{\mathrm{T}},
\tag{6}
$$

$$
Y_{i,j} := \begin{cases} 0 & ; \text{if } (s(\boldsymbol{j}_m) = 0) \\ \frac{y_{l,m}(\theta, \phi)R(\vartheta)}{d^2} & ; \text{otherwise} \end{cases},
\tag{7}
$$

where t is the number of discretized region on $\mathcal{L}$, $Y_{i,j}$ is an element of matrix $\boldsymbol{Y}$, $R(\vartheta)$ is a reflectance distribution function, ϑ is determined by the angle between a normal of the board and the ray, and d is the distance between (u, v) and $\boldsymbol{X}$ on $\mathcal{M}$.

IV. EXPERIMENT

To validate the effectiveness of the proposed method, we estimate the 4-D light field using the synthesized and real data set. For all experiments, we used a calibrated camera with a resolution of 480×360 pixels with 16-bit color depth. With a fixed camera and light source, we captured multiple images of the Lambertian board at various distances from the camera.
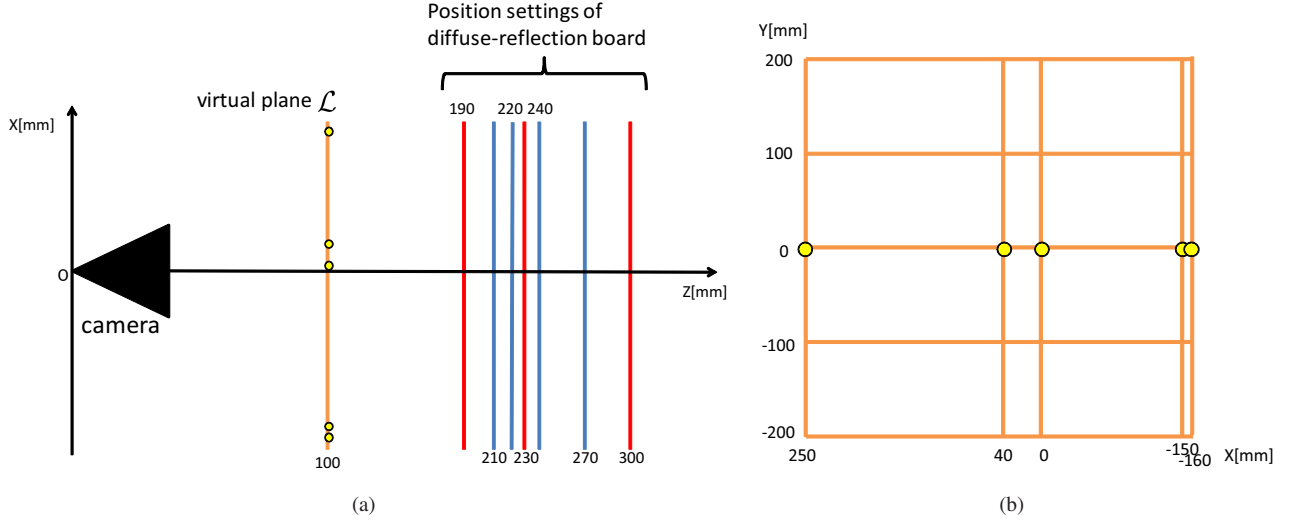
Fig. 4. Experimental settings (a) layout of camera, light source, and diffuse board. (b) positions of directional light sources on virtual plane $\mathcal{L}$. The blue and red lines indicate the positions of diffuse-reflection board for captured images. Only images for blue positions are used for input and the others are used for comparison. The yellow circles indicate the position of the directional light sources.
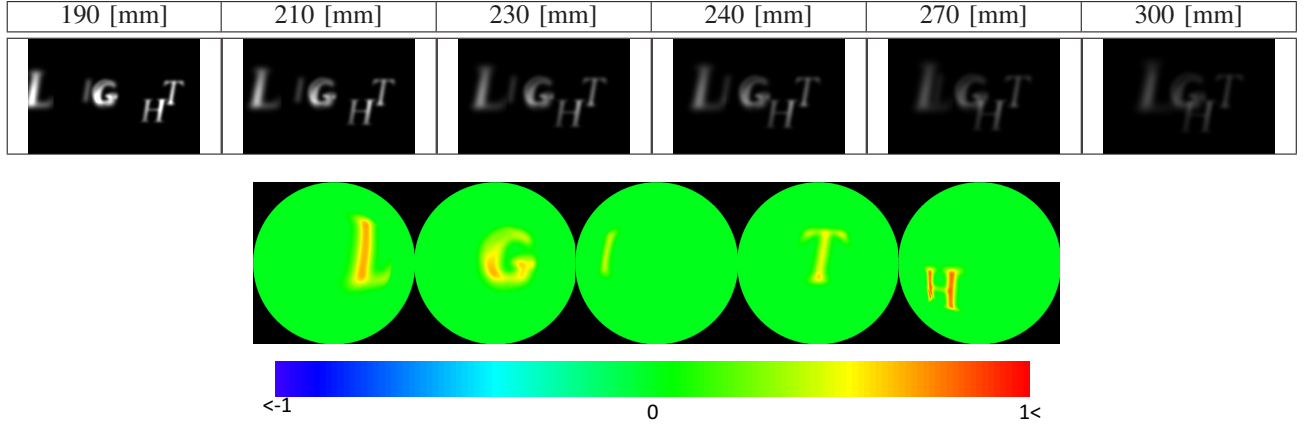

Fig. 5. Captured images and given light field map for generating input images. Radiance of light field map is normalized to 0 - 1.

A. Light field estimation in a simulation

In this experiment, we estimate the 4-D illumination light field using the simulation environment. Figure 4 (a) shows the setup of a camera, light sources and a board in this experiment. As input, four out of seven images are used. From the 3-D position of yellow circles in Fig. 4 (b), strong directional lights, which are five alphabets 'L-G-I-H-T', are projected to the board so that they can be read as 'L-I-G-H-T' with different disparity on the projected images as shown in Fig. 5. By using these synthesized images, we have estimated the light field for the position of grid points shown in Fig. 5. Figure 6 compares relighted images using estimated light field with the following three methods.

(i)   4-D light field estimation with visibility constraint,

(ii) 4-D light field estimation without visibility constraint,

(iii) 2-D light field estimation without visibility constraint.

For the method (iii), coefficient parameters c of the real spherical harmonics function are estimated for the 3-D position (250, 0, 100). Except for this, common parameters are used

and dimension k of the real spherical harmonics function is set to 441 in this experiment. By comparing generated images and ground truth shown in Figs. 5 and 6, we can confirm that relighting images generated by the proposed method (i) are the best among the compared methods others. As we can see (ii) in this figure, relighted images without visibility constraints are blurred and the words are unreadable for 190mm and 300mm, which are outside of the depth range of the board for input images. This is caused by the shortage of the available parameters for the real spherical harmonics to adjust all the observed intensities. As we can see in (iii), 2-D light field cannot produce good relighting images for complex light field.

Each circle in Fig. 6 shows estimated light field map for each position. From the results for the methods (i) and (ii), we can confirm that estimated radiance intensities in (i) for each position and direction are more accurate than that in (ii).

B. Light field estimation in real scene

We have also compared three methods in real environment. In this experiment, the flashlight shown in Fig.1 is used as a

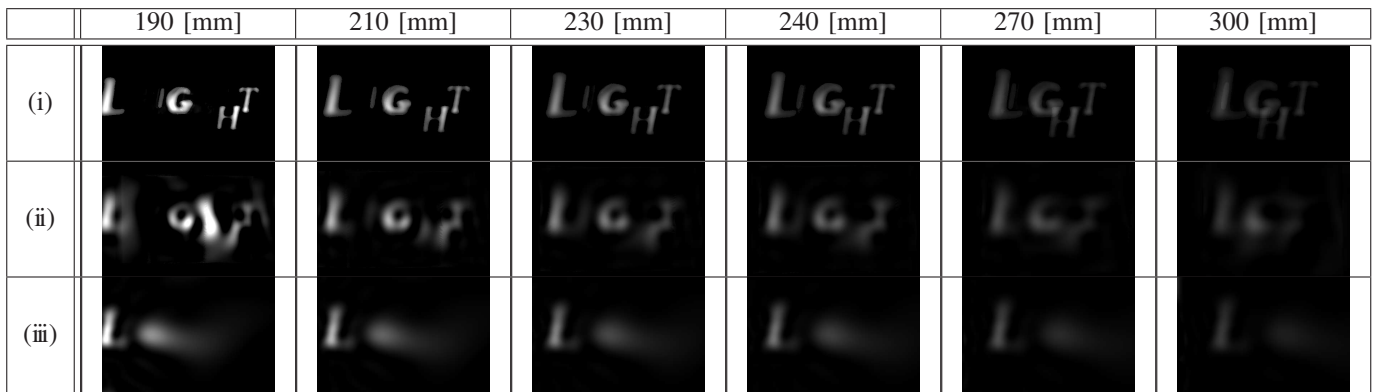|	190 [mm]	210 [mm]	230 [mm]	240 [mm]	270 [mm]	300 [mm]
(i)						
(ii)						
(iii)						

Fig. 6. Relighted images by estimated light fields from compared methods. (i): 4-D light field estimation with visibility constraint. (ii) 4-D light field estimation. (iii) 2-D light field estimation.
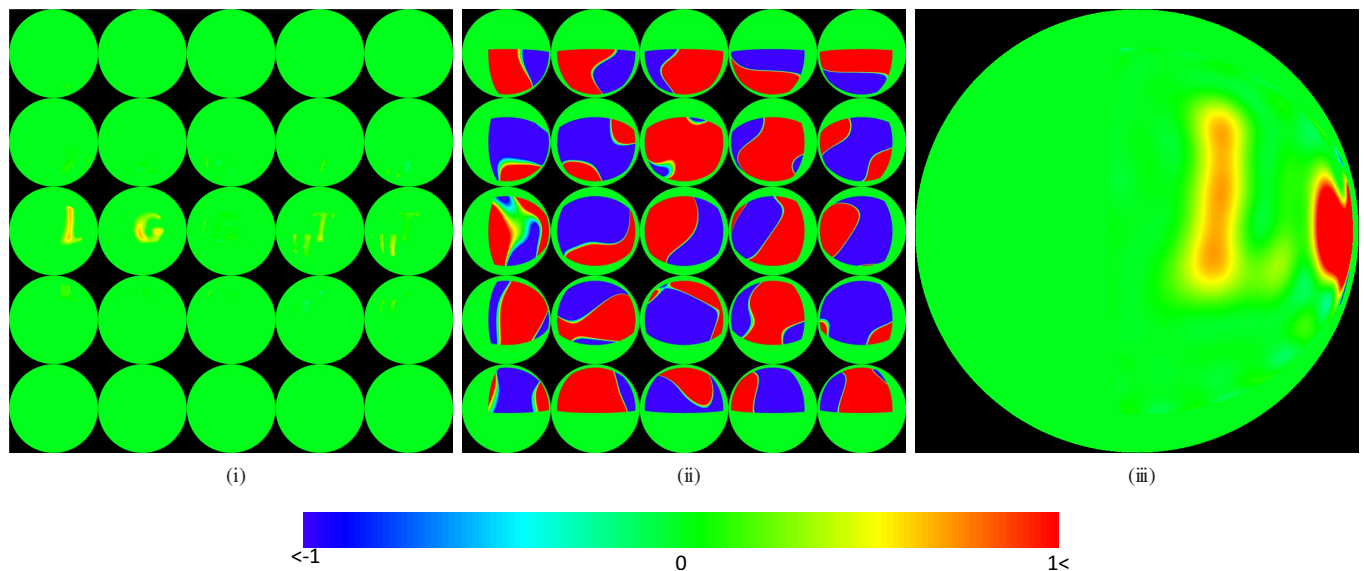

Fig. 7. Output light field map. (i)-(ii) correspond Fig. 6. Estimated light field maps for each grid point.

light source. Relative pose of the board and the camera is estimated using attached markers.

Figure 8 shows the captured images where the board is set at different distances from the camera (400, 500, 600, 700mm). In this experiment, we set a virtual plane $\mathcal{L}$ in front of the flashlight and we have estimated light map for grid points of 5×5 regular grid where the size of each grid is 20mm×20mm. Figure 9 shows relighted images using estimated light field for the compared methods. Fig. 10 shows estimated light field map with $k = 121$. As we can confirm from these figures, the proposed method works even for complex light field in real environment, and visibility mask is very important to acquire stable result. However, we can see that there exist many negative values in estimated light field in this figure. It is considered to be appeared due to the lack of valid observations for solving linear system. A solution for removing negative values should be investigated in future work for estimating 4-D light field more accurately.

V. CONCLUSION

In this paper, we have proposed a method for estimating 4-D illumination light field defined for one complex light source.

Unlike conventional approaches, the proposed method does not require a huge number of images for direct observation of light sources to estimate 4-D light field as shown in experiments. Our approach decompose the observed intensities on diffuse-reflection board into radiance intensities of light rays with linear solution. We have introduced spherical harmonics and visibility mask for stable solution of the system and their effectiveness is validated through the experiments. Estimated 4-D light field will be used to develop a more practical 3-D reconstruction technique with photometric stereo in next stage.

REFERENCES

[1] T. Aoto, T. Taketomi, T. Sato, Y. Mukaigawa, and N. Yokoya, "Position estimation of near point light sources using clear hollow sphere," in *Proc. IAPR ICPR2012*, 2012, pp. 3721–3724.
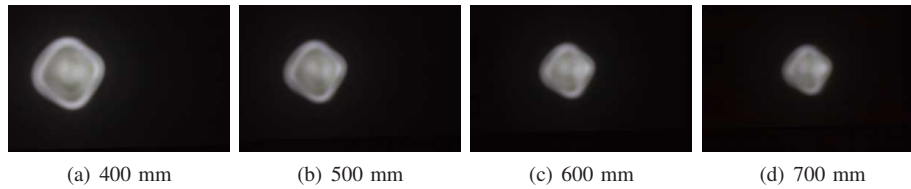
	400 [mm]	500 [mm]	600 [mm]	700 [mm]
(i)				
(ii)				
(iii)				

Fig. 8.   Captured images in a real situation.

(a) 400 mm   (b) 500 mm   (c) 600 mm   (d) 700 mm
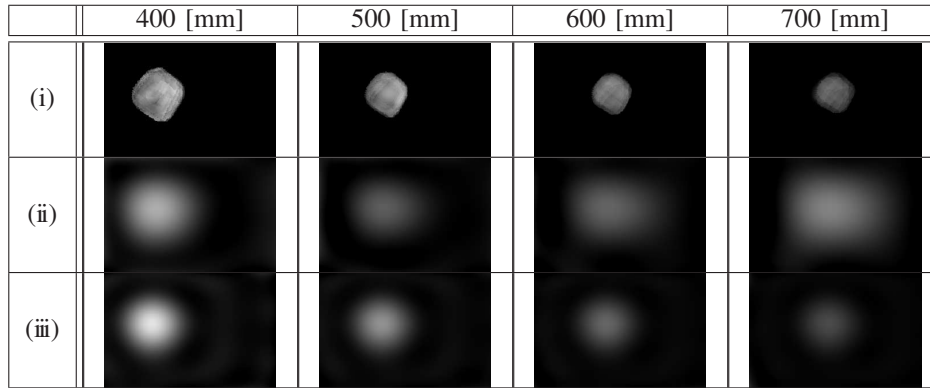
Fig. 9.   Relighting images by estimated light fields from compared method in real scene. (i): 4-D light field estimation with visibility constraint. (ii) 4-D light field estimation. (iii) 2-D light field estimation.
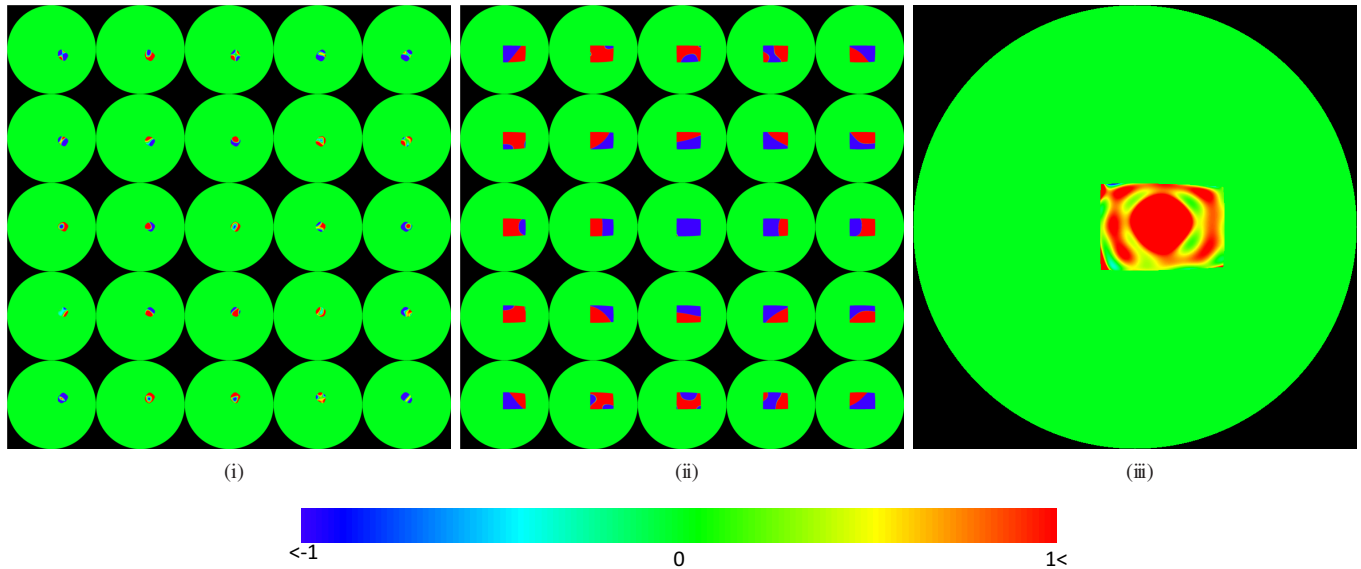

(i)   (ii)   (iii)

<-1   0   1<

Fig. 10.   Estimated light field map for each grid point in real scene.

[2] T. Takai, S. Nimura, A. Maki, and T. Matsuyama, "Self shadows and cast shadows in estimating illumination distribution," in *Proc. European Conf. on Visual Media Production*, 2007, pp. 1–10.

[3] C. Verbeck and D. GreenBerg, "A comprehensive light source description for computer graphics," *IEEE Computer Graphics and Applications*, vol. 4, no. 7, pp. 66–75, 1984.

[4] P. Debevec and J. Malik, "Recoverring high dynamic range radiance maps from photo graphs," in *Proc. ACM SIGGRAPH*, 2008, pp. 369–378.

[5] I. Ashdown, "Near-field photometry : A new approach," *Journal of the Illuminating Society*, vol. 22, no. 1, pp. 163–180, 1993.

[6] M. W. Seigel and R. D. Stock, "A general near-zone light source model and its application to computer automated reflector design," *SPIE Optical Engineering*, vol. 35, no. 9, pp. 2661–2679, 1996.

[7] R. F. Rykowski and C. Wooley, "Source modeling for illumination dsign," *In Lens Design, Illumination, and Optomechanical Modeling, SPIE*, vol. 3130, pp. 204–208, 1997.

[8] D. R. Jenkins and H. Monch, "Source imaging goniometer method of light source characterization for accurate projection system design," in *Proc. of SID Conf.*, no. 862–865, 2000.

[9] J. Unger, A. Wenger, T. Hawkins, A. Gardner, and P. Debevec, "Capturing and rendering with incident light fields," in *Proc. Eurographics symposium on rendering*, 2003, pp. 141–149.

[10] O. Cossairt, S. K. Nayar, and R. Ramamoorthi, "Light field transfer: Global illumination between real and synthetic objects," *ACM Trans. on Graphics*, vol. 27, no. 3, pp. 57:1–57:6, 2008.

[11] M. Goesele, X. Granier, W. Heidrich, and H.-P. Seidel, "Accurate lightsource acquisition and rendering," in *Proc. ACM SIGGRAPH*, 2003, pp. 621–630.

[12] I. Sato, Y. Sato, and K. Ikeuchi, "Illumination from shadows," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 25, no. 3, pp. 290–300, 2003.