Symplectic Raytracing:
Raytracing with Hamiltonian Dynamics in Blackhole Spacetime

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Introduction

This technical sketch presents *symplectic raytracing*, which is a novel approach to extend raytracing in the curved spacetime with blackholes. In conventional study for visualizing blackhole spacetime, the path of light is computed by solving geodesic equations numerically. However, raytracing based on the geodesic equation suffers from some problems concerning the computational cost and the accuracy of results. In order to overcome such problems, we have developed *symplectic raytracing* based on Hamilton's canonical equation instead of the geodesic equation. Hamilton's canonical equation can be numerically solved by a symplectic solver suited to long-time computation in blackhole spacetime.

Symplectic Raytracing

Original raytracing technique [1] traces the light projected from light sources (or a viewpoint) assuming that the path of light draws a straight line. An equation of straight line is given by:

$$\frac{d^2x^i}{ds^2} = 0, \tag{1}$$

where x^i indicates the component of three dimensional coordinates and s is a parameter of the straight line. In gravitational raytracing [2] extended from original raytracing, the following geodesic equation is employed to compute a path of light:

$$\frac{d^2x^i}{ds^2} + \Gamma^i_{kl}\frac{dx^k}{ds}\frac{dx^l}{ds} = 0, \tag{2}$$

where Γ^i_{kl}, called Christoffel's symbol, is a function to calculate curvature of space and other variables are the same as in Eq. (1). By adding the term of Γ^i_{kl} to Eq. (1), Eq. (2) have some difficulties as follows. Firstly, Eq. (2) is a second order nonlinear differential equation of parameter s. Secondly, any suitable numerical method for solving Eq. (2) have been not proposed. Thirdly, it is not easy to concretely derive the geodesic equation of blackhole spacetime.

From the above consideration, we propose *symplectic raytracing*, which is a new raytracing method using the following Hamilton's canonical equation instead of the geodesic equation.

$$\left\{ \begin{array}{rcl} \dfrac{dp_i}{ds} & = & -\dfrac{\partial H}{\partial q_i} \\ \dfrac{dq_i}{ds} & = & \dfrac{\partial H}{\partial p_i} \end{array} \right. , \tag{3}$$

where q_i indicates the component of four dimensional coordinates (the same as x^i in Eqs. (1) and (2)), p_i indicates the momentum of q_i , and H called Hamiltonian is a function of q_i and p_i. Note that symplectic raytracing needs eight dimensional phase-space constructed of four coordinate components and four momentum components. Compared with Eq. (2), Eq. (3) is a first order linear differential equation so that the suitable numerical method called *symplectic numerical analysis* [3] is applicable and the concrete derivation of Hamilton's canonical equation for the blackhole spacetime is simple. Especially, it is important to introduce symplectic numerical analysis. Because non-symplectic numerical analyses such as classical Runge-Kutta methods and Euler method are known to break the energy conservation law, they are not suitable for long-time calculation to trace the light in the universe. The main difference between the proposed and conventional methods for raytracing in curved spacetime is whether symplectic numerical analysis is applicable or not.

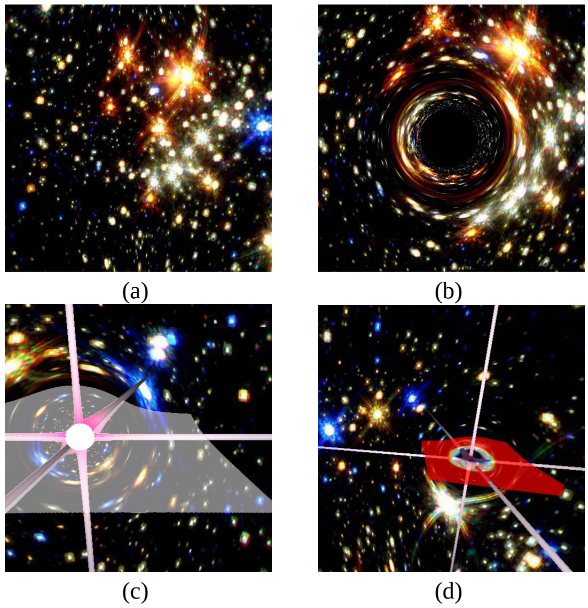

Figure 1: Visualization of a spherically symmetric blackhole; (a) no blackhole, (b) blackhole in the center, (c) superimposing x-y-z axes and x-y plane, (d) volume rendering of scalar curvature as well as three axes.

Visualization of Blackhole Spacetime

We assume the universe can be modeled by sphere and a blackhole is located at the center of the sphere. An image of galaxies[1] is mapped on the inside surface of the sphere. An observer is in the celestial sphere. Images perceived by the observer are generated with the proposed method as shown in Fig.1.

Figure 2: Immersive environment for observing a blackhole spacetime.

Because symplectic raytracing is specialized for Hamilton system, precisely and fast computation is possible more than conventional raytracing with the geodesic equation. All images in Fig. 1 are rendered on the SGI Onyx2 and Origin2000 with parallel computation by OpenMP. Each rendering took thirty to sixty minutes using 13 MIPS R10000 CPUs. We have also implemented the method on the cylindrical immersive projection display CYLINDRA(See Fig. 2).

References

[1] R. A. Goldstein and R. Nagel. 3-d visual simulation. *Simulation*, 23(6):25–31, 1971.

[2] Y. Yamashita. Computer graphics of black holes: The extension of raytracings to 4-dimensional curved space-time. *Trans. Information Processing Society of Japan*, 30(5):642–651, 1989. (in Japanese).

[3] H. Yoshida. Recent progress in the theory and application of symplectic integrators. *Celestial Mechanics and Dynamical Astronomy*, 56:27–43, 1993.

[1] http://oposite.stsci.edu/pubinfo/pr/1999/30/index.html